Exam Advanced Logic

June 16th, 2015

Instructions:

- Put your name and student number on the first page.
- Put your name on subsequent pages as well.
- Do not use pencil (or an erasable pen) or a red pen to make your exam.
- Motivate all your answers.
- If you want to inspect your exam after it has been graded, you can do so by making an appointment with Rineke Verbrugge.
- Please fill in the anonymous course evaluation.

Good luck!

1. **Induction (10 pt)** Let $\Pi(A)$ be the set of propositional atoms occurring in A. For example, $\Pi((p \equiv \neg q)) = \{p, q\}$.

   Now consider the following sublanguage $\mathscr{L}_E$ of the language of propositional logic.

   i Each propositional parameter p is a formula of $\mathscr{L}_E$.

   ii If A is a formula of $\mathscr{L}_E$, then so is $\neg A$.

   iii If A and B are formulas of $\mathscr{L}_E$ such that $\Pi(A) \cap \Pi(B) = \emptyset$, then $(A \equiv B)$ is also a formula of $\mathscr{L}_E$.

   iv Nothing is a formula of $\mathscr{L}_E$ unless it is generated by finitely many repeated applications of i, ii and iii.

   Prove the following by induction:
   For each formula A of $\mathscr{L}_E$, both A and $\neg A$ are satisfiable.
   (Reminder: A formula A is satisfiable if and only if there exists a valuation $v : P \to \{0, 1\}$ such that $v(A) = 1$.)

2. **Three-valued logics (10 pt)** Using a truth table, determine whether the following inference holds in **LP**:

   $$p \supset q, \neg p \supset q \models_{LP} q$$

   Do not forget to draw a conclusion.

3. **Tableaus for FDE and related many-valued logics (10 pt)** By constructing a suitable tableau, determine whether the following inference is valid in $\mathbf{K_3}$. If the inference is invalid, provide a counter-model.

   $$\neg(p \wedge q) \vee (\neg p \wedge \neg q) \vdash_{K_3} \neg p$$

   NB: Do not forget to draw a conclusion from the tableau.

4. **Fuzzy logic (10 pt)** Determine whether the following holds in the fuzzy logic with $D = \{x : x \geq 0.7\}$. If so, show that if the premises have value at least 0.7, so does the conclusion. If not, provide a counter-model.

   $$p \to q, q \to r \models_{0.7} p \to r$$

5. **Basic modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following is valid in K. If the inference is invalid, provide a counter-model.

   $$\Box p \supset \Box(\Box p \wedge p) \vdash_K \Box p \supset \Box\Box p$$

   NB: Do not forget to draw a conclusion from the tableau.

6. **Normal modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following tense-logical inference is valid in $K^t_{\eta\delta}$. If the inference is invalid, provide a counter-model.

$$[F]p \vdash_{K^t_{\eta\delta}} \langle F\rangle\langle F\rangle p$$

NB: Do not forget to draw a conclusion from the tableau.

7. **Soundness and completeness (10pt)** The following question is about the completeness lemma for the normal modal logic $K_{\rho\tau}$.

Let b be a complete open branch of a $K_{\rho\tau}$-tableau, and let $I = \langle W, R, v\rangle$ be the interpretation that is *induced* by b. Show that the accessibility relation R of I is reflexive and transitive.

8. **First-order modal tableau, variable domain (10 pt)** By constructing a suitable tableau, determine whether the following is valid in VK. If the inference is invalid, provide a counter-model.

$$\exists xPx \supset \exists x\Box Px \vdash_{VK} \exists xPx \supset \Box\exists xPx$$

NB: Do not forget to draw a conclusion from the tableau.

9. **Default logic (10 pt)** The following translation key is given:

$G(x)$	x missed the first three lectures of Advanced Logic
$B(x)$	x became a member of the board of Cover
$A(x)$	x joined the excursion to New Zealand
d	Douwe

Consider the following set of default rules:

$$D = \left\{ \delta_1 = \frac{B(x) : \neg A(x)}{\neg A(x)}, \qquad \delta_2 = \frac{A(x) : G(x)}{B(x)}, \qquad \delta_3 = \frac{G(x) : A(x)}{A(x)} \right\},$$

and initial set of facts:

$$W = \{B(d), G(d)\}.$$

This exercise is about the default theory $T = (W, D)$; so you only need to apply the defaults to the relevant constant d.

(a) Of each of the following sequences, state whether it is a *process*; and if so, whether or not the process is *closed*, and whether or not it is *successful*. Briefly explain your answers.

i. $\emptyset$ (the empty sequence ( ))
ii. (δ_1)
iii. (δ_2)
iv. (δ_3)
v. (δ_3, δ_2)
vi. $(\delta_3, \delta_2, \delta_1)$

(b) Draw the process tree of the default theory (W, D).

(c) What are the extensions of (W, D)?